Let = the REML-estimator. Show that the following properties hold:

1. *E[ =*
2. *Cov( =*
3. *E[ =*

Solution:

i)

The LS-estimator may be written as follows:

[For linear regression model Yi = , where I = 1, 2, …., n]

We can now get the expectation vector and the covariance matrix from the LS-estimator:

[

ii)

Cov( …………………………. (1)

Now,

Now from equation (1) [2],

*Cov*(*E[(*

[

iii)

we know,

M is a (deterministic) symmetric and idempotent matrix; Hence, we can write:

Also, obtain a quadratic form in , with other words a scalar. With the help of the trace operator *tr* we obtain [1],

*E*

[

*[use tr(XY) = tr(YX)]*

*[use tr(X+Y) = tr(Y) + tr(X)]*

[The matrix *In – H* is also symmetric and idempotent with *rk(In – H) = n – p – 1*]

Hence,

*E[ = =*

1. <https://www.fm.mathematik.uni-muenchen.de/teaching/teaching_ss15/lectures/regression/notes.pdf>
2. Proofs involving ordinary least squares, https://en.wikipedia.org/wiki/Proofs\_involving\_ordinary\_least\_squares